Ladder Circuit Modeling of Dynamic Hysteretic Property Representing Excess Eddy-Current Loss

Itsuki Suehiro[1], Takeshi Mifune[1], *Member, IEEE*, Tetsuji Matsuo[1], *Member, IEEE*, Junji Kitao[2], Taiga Komatsu[2], and Masatsugu Nakano[2]

[1]Graduate School of Engineering, Kyoto University, Kyoto, 615-8510 Japan, matsuo.tetsuji.5u@kyoto-u.ac.jp
[2]Mitsubishi Electric Corporation, Hyogo, 661-8661 Japan, Kitao.Junji@df.mitsubishielectric.co.jp

This paper combines Bertotti's model for the excess eddy-current loss with the dynamic hysteresis model represented by the Cauer circuit and the play model. The excess eddy-current field is represented by a nonlinear resistance, which improves the AC iron-loss evaluation up to 20kHz under sinusoidal excitation and achieves accurate BH loop representation under PWM excitation.

***Index Terms*—Cauer cicuit, excess eddy-current loss, hysteresis, PWM, silicon steel.**

I. Introduction

Silicon steel sheets are widely used for iron-core material, which have complex magnetic properties such as hysteresis, anisotropy and excess eddy-current loss [1]. The PWM excitation is generally used for the motor drive control, which yields minor hysteresis loops because of higher harmonic components. In order to describe the dynamic hysteretic property, the play model [2] was combined with the Cauer circuit [3], which successfully represented AC hysteretic property [4]. However, the accurate iron-loss evaluation in wide frequency range requires an accurate description of excess eddy-current loss.

To model the excess eddy-current loss, Bertotti's theory [1] is often used in the magnetic field simulation. Ref. [5], however, reported that its simple application does not achieve good accuracy in high frequency range. To improve the modeling accuracy of dynamic hysteresis, this paper discusses the combination of Bertotti's theory with the Cauer circuit and the play model.

II. Cauer circuit and Bertotti's theory

A. Circuit representation of excess eddy-current field

The Cauer circuit is a ladder network as shown in Fig. 1. The terminal current i corresponds to the surface magnetic field $\mathrm{H_s}$, and the terminal voltage v corresponds to the time derivative of average magnetic flux density of silicon steel sheet.

Using Bertotti's theory for excess eddy-current loss, the magnetic field is described as

$$H = H_{DC}(B) + \frac{\sigma d^2}{12}\frac{\mathrm{d}B}{\mathrm{d}t} + C_{\mathrm{brt}}\left(\frac{dB}{dt}\right)^{\frac{1}{2}} \tag{1}$$

where $H_{\mathrm{DC}}(\mathrm{B})$is the DC magnetic field, σ is the conductivity, d is the sheet thickness and C_{brt} is a constant for the excess eddy-current field. The circuit representation of (1) is shown in Fig. 2. In this circuit, the inductor L represents the DC magnetic field, the resistor $3R = 12/(\sigma d^2)$ represents the classical eddy-current field and the non-linear resistor C_{brt} represents the excess eddy-current field. The current-voltage property of nonlinear resistor is given as $i = C_{\mathrm{brt}} v^{1/2}$.

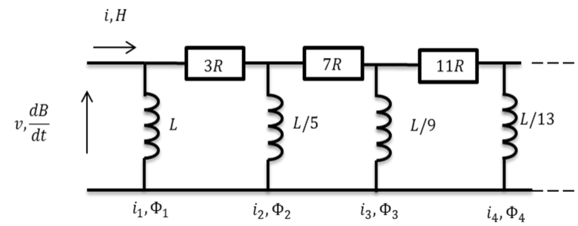

Fig.1 Cauer Circuit

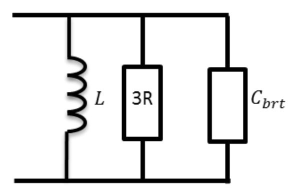

Fig.2 Circuit representation of excess eddy current field.

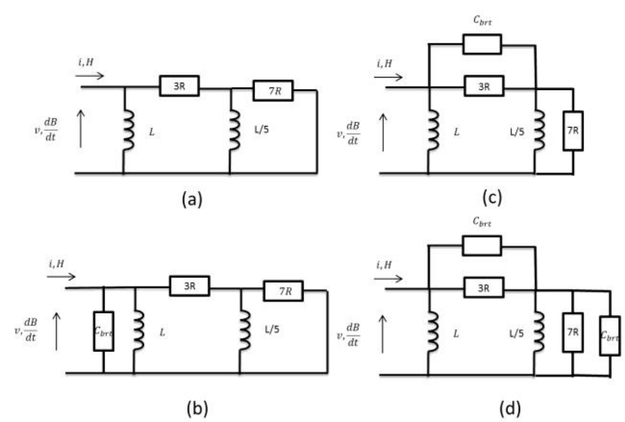

Fig.3 Cauer circuit (a) without and (b)-(d) with Bertotti's excess eddy-current field

In this study, the Cauer circuit is terminated with two inductors and resistors as shown in Fig. 3(a). The nonlinear resistor C_{brt} is added to the truncated Cauer circuit as shown in Fig. 3(b)-(d).

B. Hysteretic inductors

The DC hysteretic property $H = H_{\mathrm{DC}}(B)$ of silicon steel is represented by the play model. In the Cauer circuit, the current of first inductor L is given as $i_1 = H_{\mathrm{DC}}(\Phi_1)$. The current of k-th inductor L/α_k is simply given as $i_k = H_{\mathrm{DC}}(\alpha_k \Phi_k)$, which is called simple approximation. The finite difference approximation [4] is also used to represent inductor L/α_k.

C. Loss separation

The iron loss P per cycle is given as $P = P_{\mathrm{hy}} + P_{\mathrm{cl}} + P_{\mathrm{ex}}$ where P_{hy} is the hysteresis loss, P_{cl} is the classic eddy-current loss and P_{ex} is the excess eddy-current loss. When a sinusoidal flux is applied, the classic eddy-current is given as

$$P_{\mathrm{cl}} = \frac{\pi^2 \sigma d^2}{6} B_{\mathrm{amp}}^2 f \tag{2}$$

where B_{amp} is the amplitude of magnetic flux density and f is the frequency. The excess eddy-current loss is given as

$$P_{\mathrm{ex}} = 1.11\sqrt{2} C_{\mathrm{brt}} \left(\pi B_{\mathrm{amp}}\right)^{\frac{3}{2}} f^{\frac{1}{2}} . \tag{3}$$

Using the frequency dependence of loss components, P_{ex} can be separated from the other components by the iron loss measurement at two frequencies to determine C_{brt}.

III. COMPUTATION RESULT

Fig. 4 and Table I compare simulated iron loss per cycle using models (a)-(d) with measured one of non-oriented silicon steel sheet where $\sigma = 1.82 \times 10^6$ S/m and $d = 0.3$ mm. Sinusoidal flux is given with the amplitude of $0.5\,\mathrm{T}$, where the frequency range is from 50 Hz to 20 kHz; $C_{\mathrm{brt}} = 0.381$ A/V$^{1/2}$ is determined from loss measurement at 50 and 100 Hz. To take account of the excess eddy-current loss in circuit (a), an anomaly factor is multiplied to the conductivity, which adjusts the simulated loss to the measured one at 50Hz. Every circuit model gives accurate iron loss under 1 kHz. Circuit models (c) and (d) with simple approximation are most accurate up to 20 kHz. Circuit model (a) using the effective conductivity and model (b) simply adding the excess eddy-current field overestimate the iron loss over 1 kHz.

Figs. 5 and 6 depict the simulated BH loops under PWM excitation with the fundamental and carrier frequencies of 200 and 5000 Hz. Complex BH loop is accurately represented by circuit model (c) with finite difference approximation whereas model (a) overestimates the eddy-current field.

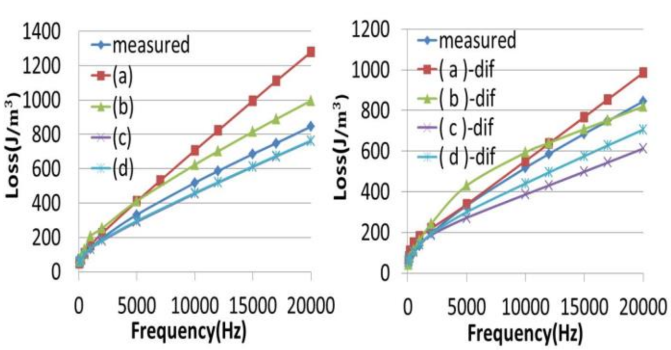

Fig.4 Frequency dependence of iron loss per cycle under sinusoidal flux condition at 0.5T: (a) simple and (b) finite difference approximations.

Table I Average of relative error

Average of relative error(%)	Simple	Finite difference
(a)	24.3	24.4
(b)	26.6	13.6
(c)	8.18	17.7
(d)	7.78	11.3

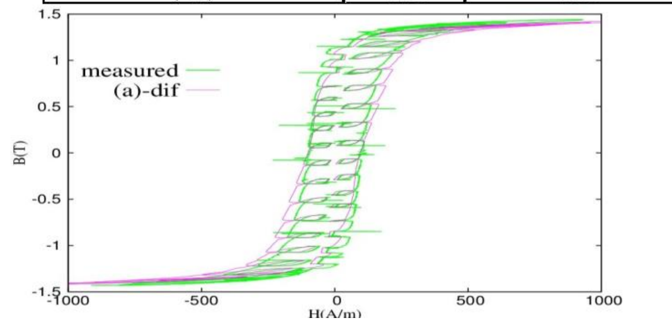

Fig. 5 BH loop under PWM excitation with circuit model (a) and finite difference approximation

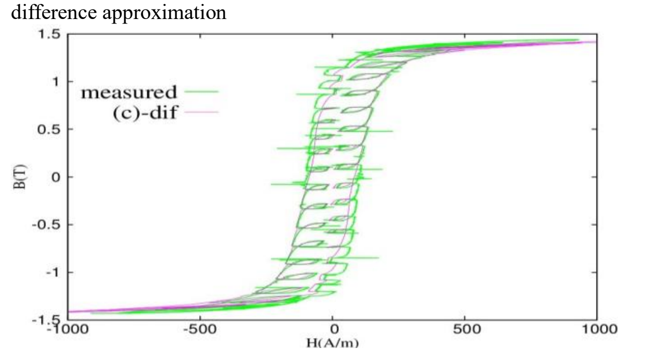

Fig.6 BH loop under PWM excitation with circuit model (c) and finite difference approximation

IV. CONCLUSION

Bertotti's theory for excess eddy-current loss is introduced to the Cauer circuit, which improves the representation of the dynamic hysteretic property under sinusoidal and PWM excitations. The effective conductivity approach or the simple addition of excess eddy-current field results in the overestimate of iron loss. Comparison with finite element eddy-current analysis will be discussed in the full paper.

ACKNOWLEDGMENT

One of the authors (T. M.) would like to express his gratitude to Dr. Y. Shindo, Kawasaki Heavy Industries, Ltd. for his advice on the Cauer circuit.

REFERENCES

[1] G. Bertotti, *Hysteresis in Magnetism*, Academic Press, 1998.

[2] T. Matsuo and M. Shimasaki, “An identification method of play model with input-dependent shape function,” *IEEE Trans. Magn.*, vol. 41, pp. 3112-3114, Oct. 2005.

[3] Y. Shindo, and O. Noro, “Simple circuit simulation models for eddy current in magnetic sheets and wires,” *IEEJ Trans. FM*, vol. 134, pp. 173-181, Apr. 2014.

[4] T. Miyazaki, T. Mifune, T. Matsuo, Y. Shindo, Y. Takahashi, and K. Fujiwara, “Equivalent circuit modeling of dynamic hysteretic property of silicon steel under pulse width modulation excitation,” *J. Appl. Phys.*, vol. 117, 17D110, 2015.

[5] K. Yamazaki and N. Fukushima, “Iron-loss modeling for rotating machines: comparison between Bertotti's three-term expression and 3-d eddy-current analysis,” *IEEE Trans. Magn.*, vol. 46, pp. 3121-3124, Aug. 2010.